Methods for Optical Components

How to handle fast oscillations in electromagnetic modeling

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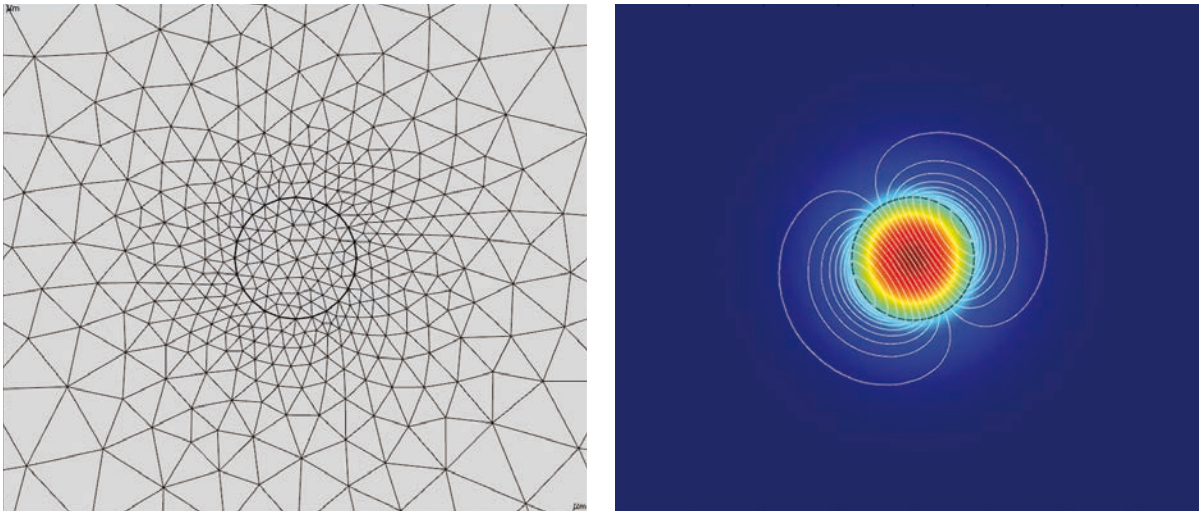

Fig. 1 A full-wave finite element analysis showing the finite element mesh (left) and one of the transverse modes of a single-mode fiber at 1.2 µm (right)

When performing computations to answer questions concerning the design of optical components, several different numerical methods can be used. Which method to choose depends on the size of the component and the nature of the electromagnetic wave propagation. Very accurate general-purpose methods, so-called full-wave methods, are frequently used for designing micro- and nano-optical devices, whereas approximate methods such as ray tracing are needed for macroscopic lens systems.

Computational high-frequency electromagnetics software utilizes an array of general-purpose numerical methods, including finite difference, spectral, method-of-moments, and finite element methods. With these tools, engineers are able to analyze propagating waves in optical structures with a minimal set of physical assumptions. Over the years, these methods have been successfully applied to the analysis of components such as optical fibers, directional couplers, and ring resonators. These general methods are sometimes referred to as full-wave methods. The name conveys that there are no inherent approximations other than those coming from creating a digital model by discretizing the piecewise continuous optical media. In this way, phenomena such as diffraction and low-order mode resonances can be captured by virtually arbitrary accuracy by simply refining the level of discretization (**Fig. 1**).

In case of the finite difference method, refining the level of discretization roughly means inserting more points in the computational domain so that the electromagnetic field can be represented in a smoother fashion, and similarly for other methods. However, more discretization points mean higher computational cost. For a 3D analysis, the number of discretization points, or elements, scale as the cube of the wavelength due to the Nyquist criterion. You need to have

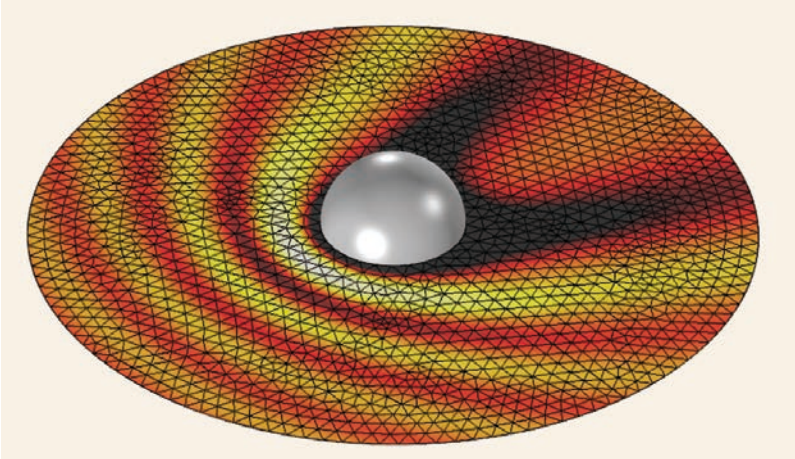

Fig. 2 A full-wave finite element analysis of electromagnetic waves that scatter off a metallic sphere. The electric field variations make it necessary to use a fine mesh throughout the computational domain. As a consequence of the Nyquist-Shannon sampling theorem, each wavelength needs to be resolved with at least two sampling points to avoid aliasing. This leads to a requirement of at least a handful of finite elements per wavelength in each spatial direction.

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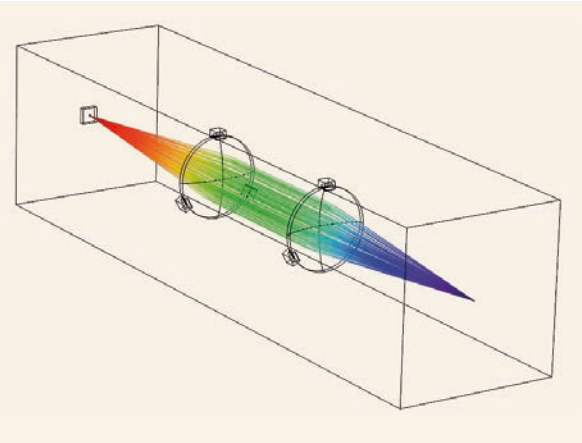

Fig. 3 For analyzing electromagnetic wave propagation in a system of lenses, the most commonly used methods are based on ray tracing.

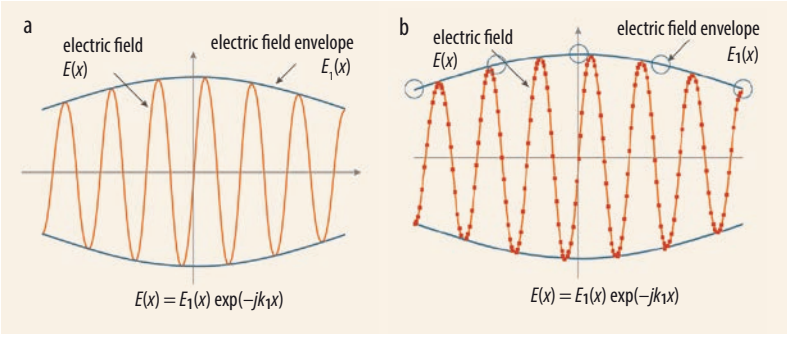

Fig. 4 (a) The beam envelope method solves the slowly varying electric field envelope E_1 by separating out the quickly varying part. After the solution for the slowly varying part is obtained by means of a computation method the full-wave field is obtained by simply multiplying the quickly varying part. (b) The beam envelope method allows for a much sparser set of sampling points than conventional full-wave methods. The reduction in the number of sample points, or nodes, can be in the orders of magnitude.

at least two sampling points per wavelength in each coordinate direction, and even more in practice. The actual computational cost of the underlying numerical methods typically scales even worse, so that full-wave methods are of limited use when the relative number of wavelengths of the object of interest is large.

For example, an optical fiber may be a few wavelengths across but many billions of wavelengths in the extruded direction. To analyze the propagating modes in the transverse direction of a cross section of a fiber, you can use one of the computationally expensive full-wave methods, since the relative size is small in this direction. However, to analyze the propagation down the fiber, including potential defects along the way, you may need to resort to approximate methods to avoid exhausting your computer's RAM (**Fig. 2**).

Approximate methods

Approximate methods like ray tracing, Gaussian optics, and beam propagation methods come with inherent simplifying assumptions. In special cases, these methods can be applied to much larger structures than full-wave approaches. For example, a centimeter-sized lens corresponds to tens of thousands of wavelengths of optical light in all directions. In this case, a ray tracing approach may be the best option. The approximations come at a cost: Using ray tracing typically implies that diffraction effects are neglected: The rays are simply traveling in straight lines (**Fig. 3**).

The beam envelope method

A guiding structure in an optical system often has a well-defined and preferred propagation direction. In the language of mathematical physics, this means that there is a well-defined wave vector that is varying slowly, or even constant, in the direction of propagation. This is utilized in a relatively new computational method called the beam envelope method, which is a full-wave method with some of the characteristics of an approximate method.

If we consider just the electrical field of a propagating wave, it has, in the most generic case, three components:

$$E = (E_x, E_y, E_z).$$

Each of the three field components can be a function of all three coordinate directions; for example, $E_x = E_x(x, y, z)$. However, if there is a preferred direction of propagation, say the z direction, then for an optically guiding component, this usually means that the field goes through many oscillations in the z direction while experiencing

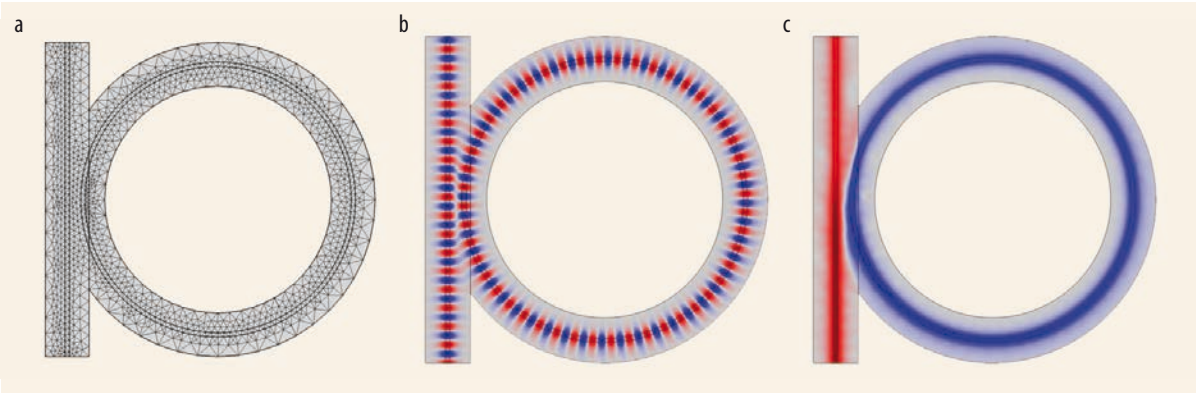

Fig. 5 A ring resonator analysis at 1.559 µm: Shown are the finite element mesh (a), the physical fast-varying field (b), and the slowly varying field envelope (c), which is the actual unknown field solved for in the beam envelope method.

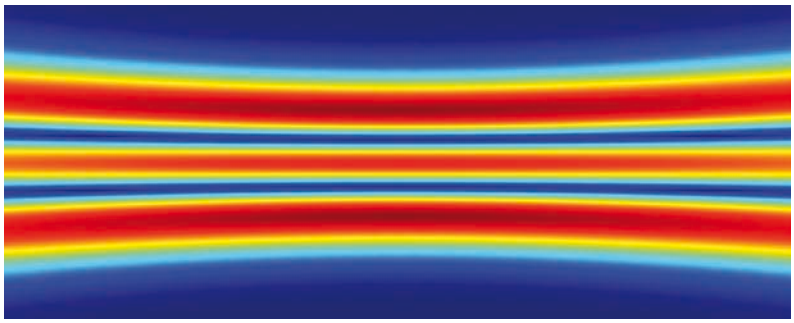

Fig. 6 Slowly varying field envelopes in a symmetric laser cavity: In this case, two propagating directions are needed for the forward- and backward-propagating waves, respectively. This is achieved by superimposing two waves in each direction.

much slower variations in the x and y directions. Therefore, for a continuous single-frequency electromagnetic wave, we may choose to write the field as

$$E = E_1 \cos(\omega t - k_z z)$$

where $\omega = 2\pi f$ is the angular frequency, k_z is the propagation constant in the z direction, z is the z coordinate, and E_1 is the slowly varying part of the field.

Note that there may still be z-variations in the slowly varying field. To clarify this, we may write it out more explicitly as

$$E(x, y, z, t) = E_1(x, y, z) \cos(\omega t - k_z z)$$

This expression, or rather its complex-valued counterpart $E = E_1 \exp[j(\omega t - k_z z)]$, can now be substituted into the full electromagnetic wave equations according to Maxwell. After some algebra, we will end up with an equation in only the slowly varying envelope field E_1. In a strike of mathematical fortune, all terms involving the quickly varying factor $\exp[j(\omega t - k_z z)]$ can be cancelled from the equation. We just need to remember to multiply with this factor after our calculation to get back the true wave representation. This fortunate cancellation of the quickly varying part of the field forms the basis of the beam envelope method. This should not be confused with the beam propagation method, which comes with additional simplifications and associated approximations resulting from throwing out some of the derivatives in the wave equation. The beam envelope method comes with no approximation but belongs to the class of full-wave methods (**Fig. 4a**).

In which way does this mathematical trick help us? It all comes down to beating the Nyquist criterion. A major obstacle when using full-wave methods is that you have to sample the field with enough computational points, or nodes. If not, your computational result will be numerical garbage. By only solving for the slowly varying envelope field, the computational points can be sampled much more sparsely; at least in cases when there is a distinct direction of propagation, such as in an optical waveguide (**Fig. 4b**).

Variable and multiple directions

Being able to analyze long slender structures with a more or less constant direction of propagation is important. It is fairly straightforward to see how to apply the beam envelope method to such cases. However, a large class of guiding structures is bent in one or more directions. Can the method also be applied in these cases? The answer is yes, if the structure is not too complicated. As long as the direction of propagation is slowly varying, the method is in good shape. To see how this works, we need to consider the full propagating field $E = E_1 \exp[j(\omega t - k_z z)]$ again. Here, the direction of propagation is hardwired to be in the z direction. In order to handle a generic direction, we need to write this instead as $E = E_1 \exp[j(\omega t - k \cdot r)]$, where

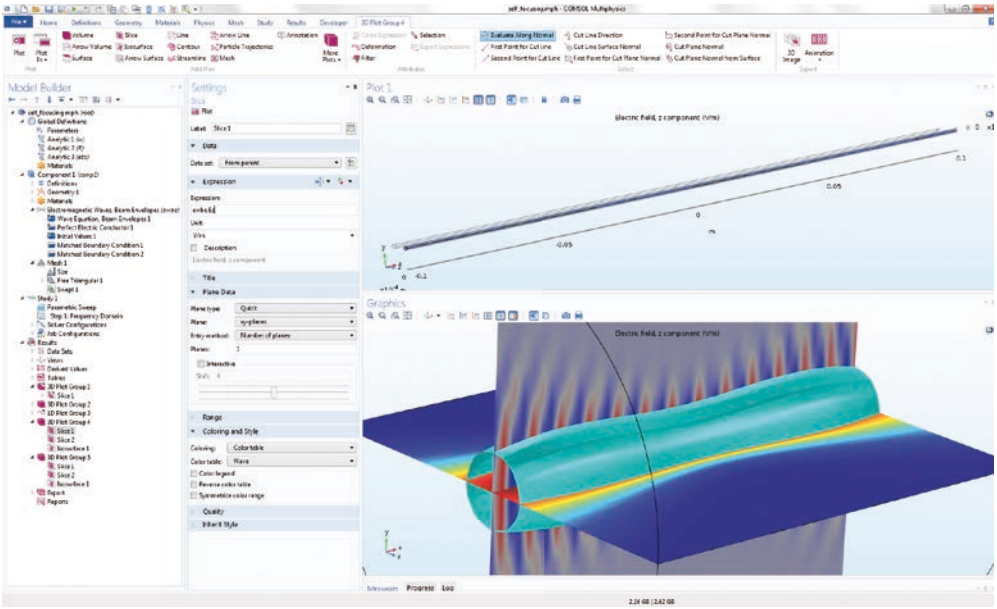

Fig. 7 A self-focusing laser beam analysis by means of the beam envelope method: The true aspect ratio representation is shown in the top panel and a compressed view in the bottom. The magnitude and z component of the slowly varying envelope electric field are shown as isosurface and slice plots. The actual physical oscillations of the electromagnetic wave are too fast to be visualized. The structure is 20 cm long and the wavelength in the propagating direction is 0.7 µm, which makes for about 300 000 wavelengths in the propagating direction.

$k = (k_x, k_y, k_z)$ is the propagation constant vector, which determines the wave's preferred direction of propagation, and $r = (x, y, z)$ is the coordinate vector.

In practice, you set the dot product $k \cdot r = k_x x + k_y y + k_z z$ equal to a spatially varying phase function $\varphi(x, y, z) = k \cdot r$. The requirement of a slowly varying direction of propagation now gets translated into a slowly varying phase. For example, the circular part of a ring resonator of radius R can be represented by a phase function $\varphi = R\,k_p \arctan(y/x)$, where $k_p = 2\pi/\lambda_p$ is the propagation constant corresponding to the wavelength λ_p in the direction of propagation.

In this way, the beam envelope method can be applied to a structure composed of simple shapes, where each component can be represented by a spatially varying phase factor corresponding to a locally preferred type of propagation. For example, a ring resonator consisting of one straight and one circular section can readily be analyzed this way by using one constant and one circular phase function. In a more general case, the phase function can be given by a look-up table that is a simple enough function of the coordinate vector. In addition, by superimposing fields, you can handle two or more directions of propagation by supplying multiple sets of phase functions (**Fig. 5** and **Fig. 6**).

Applications in nonlinear optics

Nonlinear optical effects are often very weak and occur over long interaction lengths. Here, the beam envelope method is very useful. Self-focusing is such a nonlinear phenomenon where the modification of the beam must be incorporated into the design. The effect may be seen, for example, in laser rods or glass components placed at a focal point. If the threshold for self-focusing is exceeded, the material is damaged. It is important to know the self-focusing threshold values for the used materials. Self-focusing occurs in dielectrics, like optical glasses and laser rod materials, such as Nd:YAG.

Other nonlinear effects for which the method is applicable include second harmonic generation, sum- and difference-frequency generation, parametric generation and amplification, and self-phase modulation (**Fig. 7**).

Conclusions and outlook

The beam envelope method extends the use of full-wave methods to previously unattainable model sizes. It fills a gap between computationally heavy, but accurate, traditional finite element/difference methods and fast-to-compute ray tracing methods. Successes within nonlinear optics show the method's applicability for real-world design tasks. In future, we will most likely see this method being combined with traditional full-wave and ray-tracing methods to reach new frontiers in computational optics.